Application of Zero-Compliance Mechanism to Vibration Isolation and Force Measurement

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Abstract

The zero-compliance mechanism is conceptually a series connection of a spring with positive stiffness (positive spring) and a spring with negative stiffness (negative spring) whose amplitude is equal to the positive one. When an object is suspended by this mechanism, the position of the object is maintained even if force acts on the object because the deflection of the positive spring is cancelled by the deflection of the negative spring. This characteristic produces some advantage over the conventional suspensions. This mechanism is firstly applied to vibration isolation to reduce both effects of ground vibration transmitted through the suspension and disturbance directly acting on the isolation object (table). Then it is applied to force measurement to achieve high-sensitivity force detection and no deflection of the point of force simultaneously.

Keywords: Zero-compliance, Negative stiffness, Vibration isolation, Force measurement, Micro force.

1. Introduction

In mechanical systems the most important physical relation is force-displacement relation [1]. A displacement- opposing force is produced by a normal mechanical spring. An external force is required to stretch or compress a spring because the normal spring always acts to return to its neutral state. The force in a stretched or compressed spring is often approximated to be proportional to the displacement. The proportional constant is called as spring constant and often denoted by k . The normal spring has a positive spring constant ($k > 0$) , which is referred to as positive spring in the following.

In combining two springs, there are two types of connection: parallel and series connections. In the parallel connection, the resultant spring constant is the sum of the two spring constants so that it is larger than each spring constant when both are positive springs. In contrast, the resultant spring constant is smaller than each spring constant in the series connection of two positive springs.

However, when one of the spring constants is negative, the resultant spring constant can be larger than the amplitude of each spring constant in the series connection. In particular, the resultant spring constant becomes infinite when the amplitude of the negative spring constant is equal to the positive one [2]. The concept of zero- compliance mechanism has come from finding out this relation. It can produce unique characteristics that have not been achieved by normal springs.

In this article, the principle of zero-compliance mechanism is explained first. Then, vibration isolation systems with this mechanism are presented [2-8]. The aim of using zero-compliance mechanism is to reduce both effects of ground vibration transmitted through the suspension and disturbance directly acting on the isolation table. Next, this mechanism is applied to force measurement [9-15]. The aim is to achieve high-sensitivity force detection without any deflection of the point of force. It is to be mentioned that the word "stiffness" is often used in the following mainly because "spring constant" is valid when the force-displacement relation is linear.

2. Zero-compliance

Mechanism Concept

First, it will be shown that infinite stiffness can be realized by connecting a normal spring with a spring that has negative stiffness. When two springs with spring constants k_1 and k_2 are connected in series, as shown by Fig. 1, the total stiffness k_c is given by

$$k_c = \frac{k_1 k_2}{k_1 + k_2} \tag{1}$$

This equation shows that when normal springs are connected the total stiffness becomes lower than the stiffness of each spring. However, if one of the springs has negative stiffness

$$k_1 = k_2 \tag{2}$$

the resultant stiffness becomes infinite:

$$|k_c| = \infty \tag{3}$$

In other words, the compliance is zero. Therefore, this suspension is referred to as zero-compliance mechanism. In the next chapter, this principle of generating zero compliance is applied to vibration isolation systems to counteract direct disturbances acting on the isolation table [2].

Negative stiffness

When a spring k is connected to a mass m, the equation of motion is given by

$$m\ddot{x}(t) + kx(t) = 0 \tag{4}$$

Where x is the displacement of the mass from the equilibrium point. When the spring is normal, that is $k > 0$, the force exerted by the spring on the mass is directed opposite to the displacement; the spring acts to return to its neutral state. If the spring constant is negative, that is $k < 0$, this system is obviously unstable because the spring acts to recede its neutral state. To achieve the zero-compliance states without instability, therefore, an active element (suspension) is introduced instead of a pure mechanical spring (passive element). The firstly introduced element was the zero-power magnetic suspension system, which is explained in the following.

Zero power magnetic suspension

Due to its power-saving properties, zero-power control has been used in magnetic suspension systems such as momentum wheels for spacecraft stabilization [16] and carrier systems in clean rooms [17]. In this form of control, a hybrid magnet consisting of an electromagnet and a permanent magnet is used. Control of the electromagnet makes steady deviation of the coil current converge to zero. As a result, the air-gap length is maintained so that the attractive force generated by the permanent magnet balances the other, static forces acting on the suspended object.

Figure 2 shows a single-degree-of-freedom-of-motion model for analysis [18]. The suspended object with a mass of m is assumed to move only in the vertical direction transnationally. The equation of motion is given by

$$m\ddot{x}(t) = k_s x(t) + k_i i(t) + f_d(t) \tag{5}$$

Where x is the displacement of the suspended object, k_s and k_i are, respectively, the gap-force and current-force coefficients, i is the control current, and f_d is the disturbance force acting on the suspended object. Zero-power control operates to accomplish

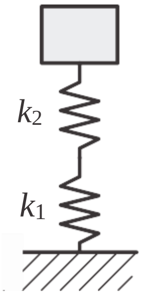


Fig.1 Series spring

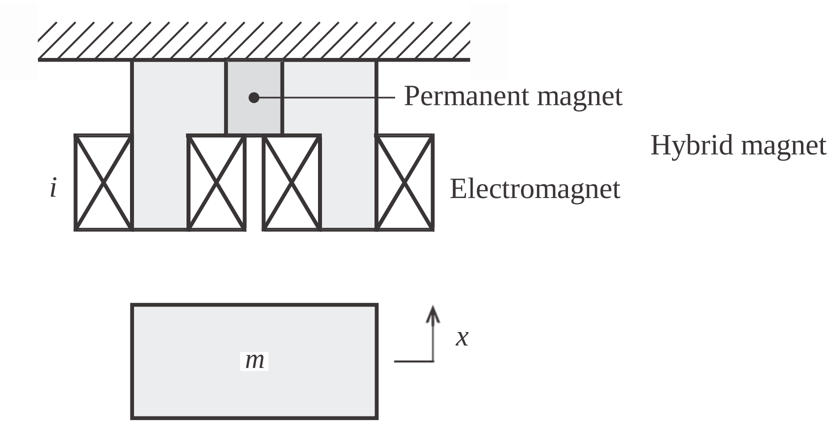


Fig.2 Model of zero-power magnetic suspension

$$\lim_{t\to\infty} i(t) = 0 \quad \text{for stepwise disturbances.} \tag{6}$$

The control input achieving zero-power control is represented by

$$I(s) = -\frac{s\tilde{h}(s)}{g(s)} X(s) \tag{7}$$

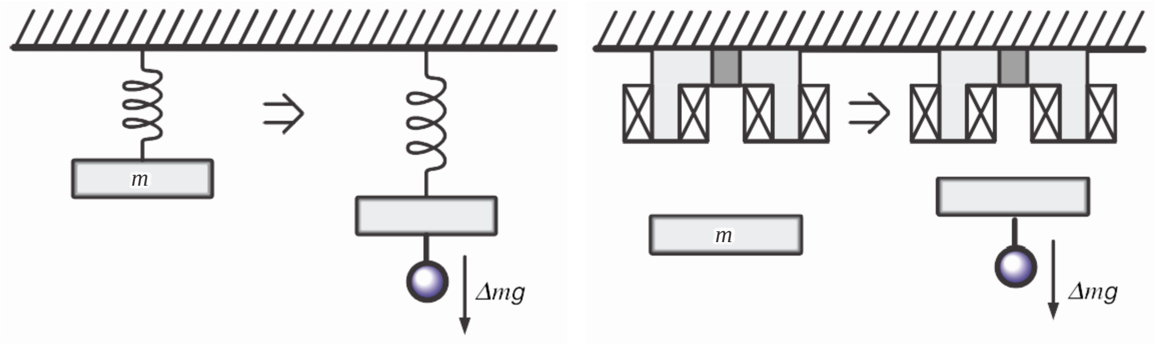
where g(s) s and $\tilde{h}(s)$ are coprime polynomials in s and selected to stabilize the closed-loop system [18].

A unique characteristic of the zero-power control system is that it behaves as if it has negative stiffness. When an external force is applied to the mass in a common mass-spring system, the mass moves to the direction of the applied force, as shown in Fig. 3a. In the system controlled by zero-power, the suspended object moves to a new equilibrium position located in the direction opposite to the applied force, as shown in Fig. 3b.

3. Vibration Isolation System

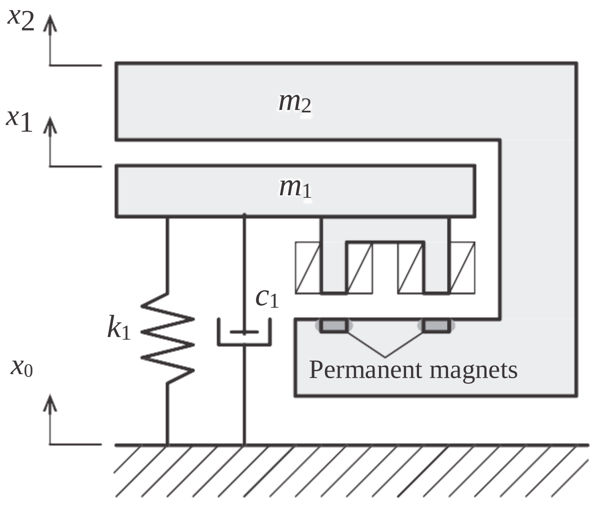
Basic configuration

Figure 4 shows the configuration of one of the proposed vibration isolation system [2]. A middle mass m_1 is connected to the base through a spring k_1 and a damper c_1, which together work as a conventional vibration isolator. An electromagnet for zero-power magnetic suspension is fixed to the middle mass. The part of an isolation table m_2 facing the electromagnet is made of a soft iron material for confining the magnetic fields produced by the permanent magnets for zero-power control. This is referred to as the reaction part.



(a) Normal spring (b) Zero-power suspension

Fig.3 Comparison of zero-power magnetic suspension system with a normal spring



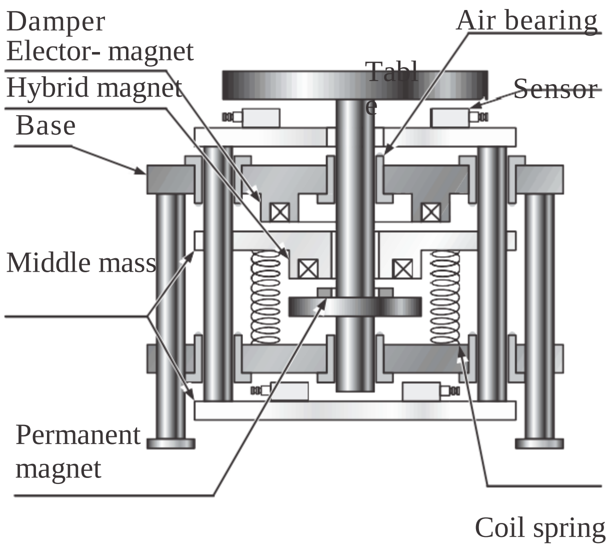


Fig.4 Model of vibration isolation system using vibration zero-power magnetic suspension

Fig,5 Schematic drawing of single-axis isolation system

This system can reduce vibration transmitted from ground by making k_1 small and, at the same time, infinite stiffness can be produced to counteract direct disturbances by setting the amplitude of negative stiffness equal to k_1 . To explain the latter more concretely: it is assumed that the table is subject to a downward force, so that the gap between the electromagnet and the table becomes smaller because of the zero-power control; that is, the table would move upwards if the middle mass were fixed. Meanwhile, the middle mass moves downwards because of the increased electromagnetic force. The decrease in the gap is cancelled by the downward displacement of the middle mass due to the above-mentioned setting. Thus, the isolation table is maintained at the same position as before.

Single-axis system

Figure 5 is a schematic diagram and a photograph of a single-axis apparatus that was built for the experimental study of the original system (Fig. 4) [4]. The height, diameter, and mass of the apparatus are 200 mm, 226 mm and 18 kg, respectively. The isolation table and the middle mass weigh 3.5 kg and 5 kg, respectively, and are guided to move transnationally in the vertical direction by linear air bearings. A ring-shape electromagnet with a 448-turn coil is fixed to the middle mass corresponding to m_1 in Fig.5; its inner and outer diameters are 68 and 138 mm, respectively. Ten 10 ×10 × 5-mm permanent magnets made of NdFeB provide bias flux. These magnets are built in the reaction part of the isolation table. This configuration widens the range of operation because repulsive force can be generated [18]. The nominal gap between the electromagnet and the permanent magnets is about 3mm. The middle mass is suspended by four mechanical springs. An electromagnet for adjusting the positive stiffness k_1 and the damping c_1 is installed on the base, and its reaction part is built in the middle mass. The electromagnet is referred to as an auxiliary electromagnet and is used to equalize the positive stiffness and the amplitude of the negative stiffness.

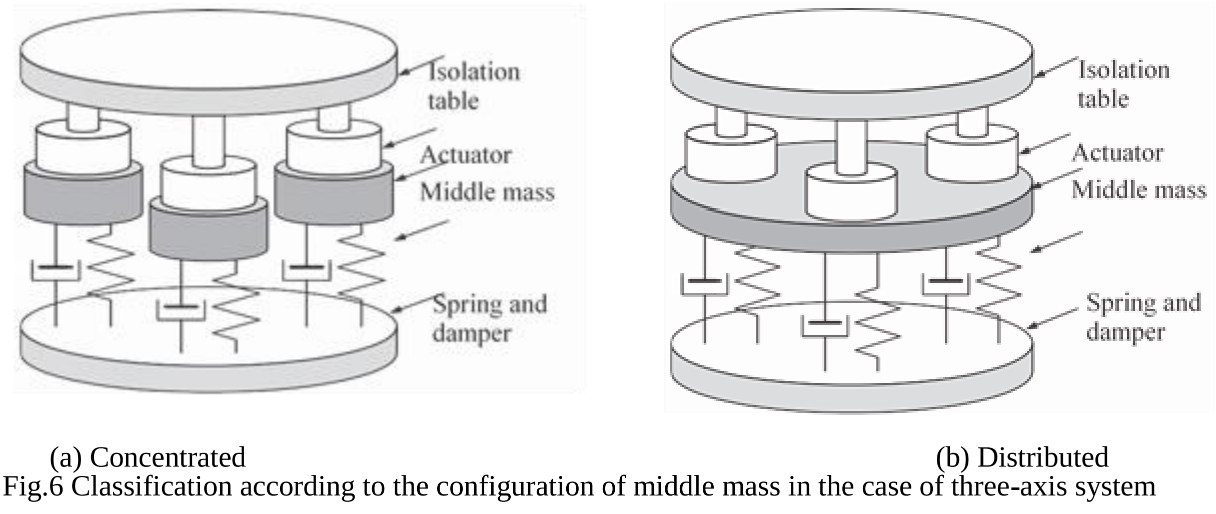
Multi-axis system

When the target motion of the isolation table is multiple, two configurations are possible as shown by Fig.6. One of them is concentrated middle mass. The other is distributed middle. One of the advantages of the former types is that it is suitable for a larger middle mass, which will be effective to reduce vibration transmitted from the ground. Figure 7 shows a picture and a schematic diagram of a three-axis vibration isolation system [5]. It is equipped with three pairs of a mechanical spring and a magnetic suspension mechanism. A translation in the vertical direction and two rotations around the horizontal axes of the isolation table are controlled with the proposed mechanism.

Meanwhile, one of the advantages of the distributed-middle-mass type is *modularization*. A multi-axis system can be constructed by combining multiple vibration isolation modules. The single-axis system shown in the previous section was originally developed as such a module.

Introduction of weight support mechanism

In the system shown by Fig.4, the entire weight of the isolation table is supported by zero-power magnetic suspension. When the isolation table is large, therefore, many permanent magnets are needed to suspend its weight, which will raise the cost of system. Another problem that can be expected in putting the proposed system to practical use is that the reaction part must be installed under the middle table (Fig.4), because the hybrid magnet can produce only an attractive force. This makes the structure rather complex.



(a) Concentrated (b) Distributed

Fig.6 Classification according to the configuration of middle mass in the case of three-axis system

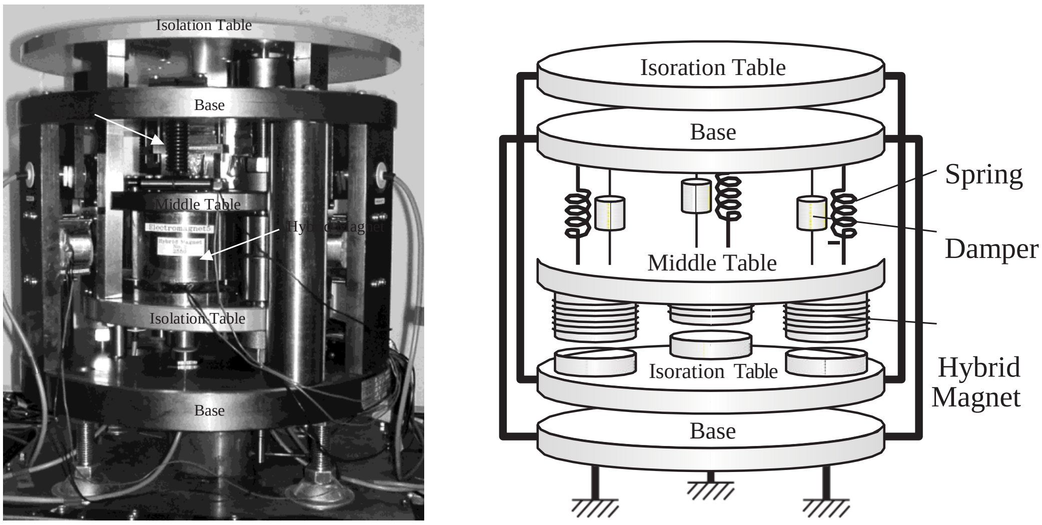
These problems can be overcome by introducing an auxiliary suspension for supporting the weight of the isolation table, as explained in Fig.8 [4]. A spring k_d is added in parallel with the serial connection of the positive and negative springs. The total stiffness k_c is given by

$$\tilde{k_c} = \frac{k_1 k_2}{k_1+k_2} + k_d \tag{10}$$

When Eq.(2) is satisfied, the resultant stiffness becomes infinite for any finite value of k_d .

$$\left|\tilde{k}_c\right| = \infty \tag{11}$$

Figure 9 shows the configuration of one of the proposed vibration isolation systems. A spring kd together with a damper c_d is inserted between the isolation table and the base. The spring is set to produce upward force in the equilibrium state. It reduces the static load force that the zero-power magnetic suspension must support. Moreover, when the upward force is greater than the gravitational force, the zero-power magnetic suspension must produce downward force so that the configuration is modified, as shown in Fig.10. Since the reaction part is installed above the middle table, the structure is simpler than the original one shown in Fig.4. It should be



(a) Picture (b) Schematic drawing

Fig.7 Three-axis vibration isolation system with a concentrated middle mass

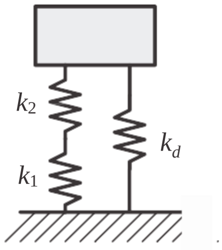


Fig.8 Parallel connection of series spring and a spring

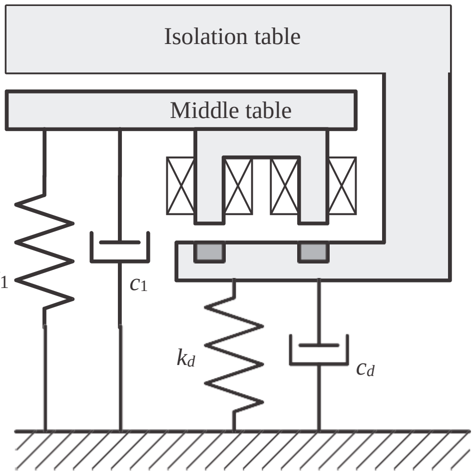
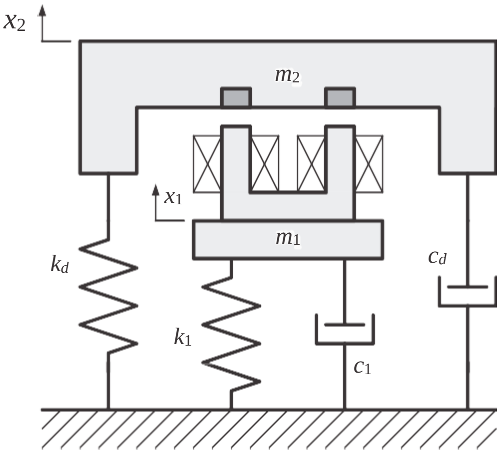


Fig.9 Modified structure with a weight support mechanism



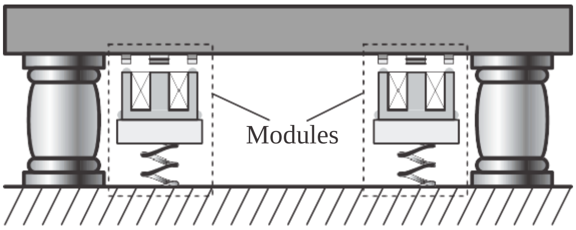


Fig.11 Retrofitted system with zero-compliance modules

Fig.10 Another modified structure with a weight support mechanism

noted that isolation from ground vibration can be maintained by using a soft spring as k_d .

Retrofit of passive vibration isolation system

Soft suspension is suitable for reducing the transmitted vibration because dynamic coupling between the vibration source and the isolation table is weakened. Therefore, air spring is mostly used to suspend the isolation table in common passive vibration isolation systems [19]. However, such soft suspension makes the system weak against direct disturbance; the vibration of the isolation table is easily induced by disturbance acting on the table.

To improve the performance to direct disturbance, the zero-compliance suspension can be combined with the conventional suspension as shown by Fig.11. The conventional suspension works as a weight support mechanism in this configuration [7].

Vibration isolation modules developed to retrofit passive systems. It is to be noted that a lot of passive vibration isolation systems have already been used in various fields including laboratories in science and factories in manufacturing. Therefore, this approach of retrofitting is promising.

4. Force measurement

Force is a fundamental physical quantity that plays a vital role in most mechanical systems. Accurate force measurements are required in many applications [20]. There are various methods of force measurement. They can be classified in several different ways. One classification is based on the structure of measurement system. One category is measurement by open-loop system that is referred to as deflection method [21]. The other is measurement by closed-loop system that is referred to as null method [21].

Most of the measurement devices belong to the former category. Many force transducers have been developed. A typical example is load cell. In such devices, higher resolution can be achieved as the stiffness of the mechanical conversion part is made lower. However, such low-stiffness mechanism causes several problems. The measurement conditions such as the distance between the force source and the point of action may change in measurement. In addition, the measurement bandwidth tends to be narrow.

The first problem can be avoided by using a measurement system with servomechanism, which is categorized into the null method. In such systems, higher feedback gains are usually necessary to widen the bandwidth of the closed-loop system. However, such high-gain system tends to suffer from noise generated in the control signal, which will worsen the resolution of measurement because the force is estimated from the control signal. To overcome such problems, force measurement using zero-compliance mechanism has been proposed.

Principle of measurement

Figure 11 shows a zero-compliance mechanism for force measurement [9]. The point of force Y is suspended by a series-connected suspension; the connection point becomes the detection point X. The stiffness of the connected suspensions, denoted by k_C, becomes infinite when Eq.(2) is satisfied. It indicates that the point of force does not move (y = 0) even if force acting on this point as if measured by the null method. In contrast, the detection point displaces proportionally to the force acting on the body as Δmg

$$x = \frac{f}{k_1} = -\frac{f}{k_2} \tag{4}$$

Therefore, the force can be estimated from the displacement as if measured by the deflection method. It is noted that high resolution is expected when low-stiffness suspensions are used.

Classification [9]

The proposed force measurement systems can be classified according to

- location of active element
- actuator for the active element
- location of positive-stiffness element
- control method of achieving zero-compliance

In practice, active control is necessary to achieve the zero-compliance characteristic without losing stability [2]). Therefore, at least one of the suspensions must be active. There are three ways of locating such active element:

- Suspension I
- Suspension II
- Both suspensions

The last one is most flexible but highest in cost. Therefore, we treat the first and second cases (I) and (II) in the following.

There is a great diversity of actuators available for the realization of active element. Possibilities suitable for measurement system are:

- Electromagnet
- Electrostatic actuator
- Voice coil motor (VCM)
- Piezoelectric actuator (PZT)

Another criterion of classification is the location of positive-stiffness element. There are two ways as shown in Fig.12:

- Suspension I
- Suspension II

The operation of mechanism in each case is illustrated by Fig.12. The detection point displaces in the same direction as the applied force in the first case (U) while opposite in the second case (L).

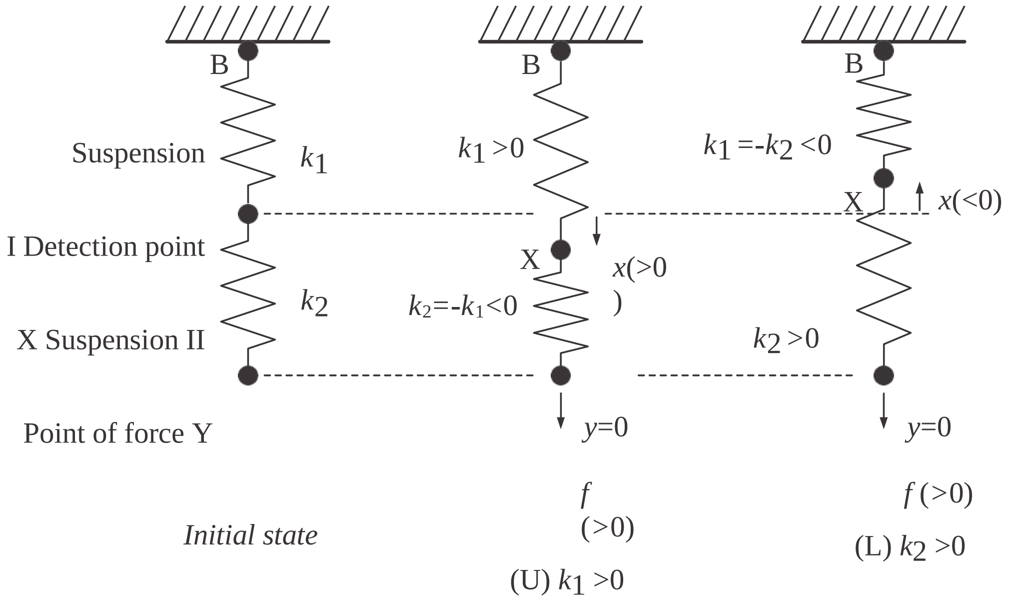
To achieve zero-compliance, there are several applicable control methods such as

- Stiffness control include proportional control and zero-power control [2, 3]
- Integral feedback of the displacement of the point of action [9, 11-18]
- Displacement cancellation control [6, 8]

Developed Apparatuses

The first apparatus used double series magnetic suspension [10] as a zero-compliance mechanism [11]. Figure 13 shows a picture and schematic drawing of a fabricated three-dimensional force measurement apparatus using double magnetic suspension [12]. Three-dimensional force measurements have been carried out to study the performances of the developed measurement system.

Another type apparatus has been also developed which uses a parallel spring and a voice coil motor as an actuator [13]. Figure 14 shows a schematic drawing of the developed measurement module. It is more suitable for miniaturization. In addition, a 3-component force measurement apparatus has been assembled with four single-axis measurement modules.

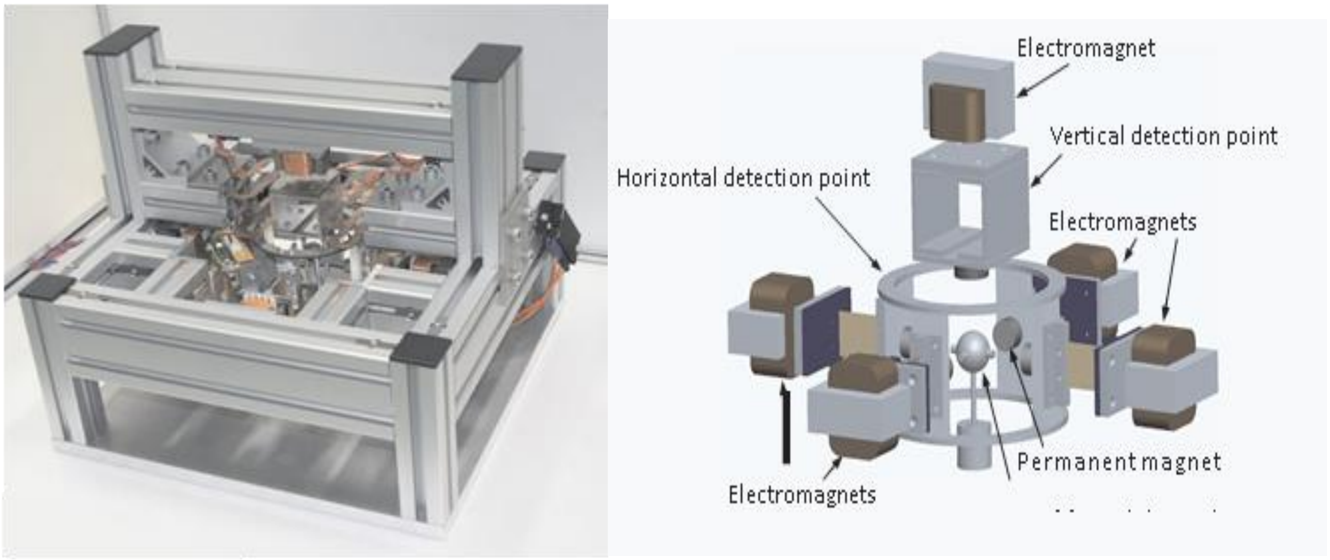


Initial state

(U) $k_1>0$

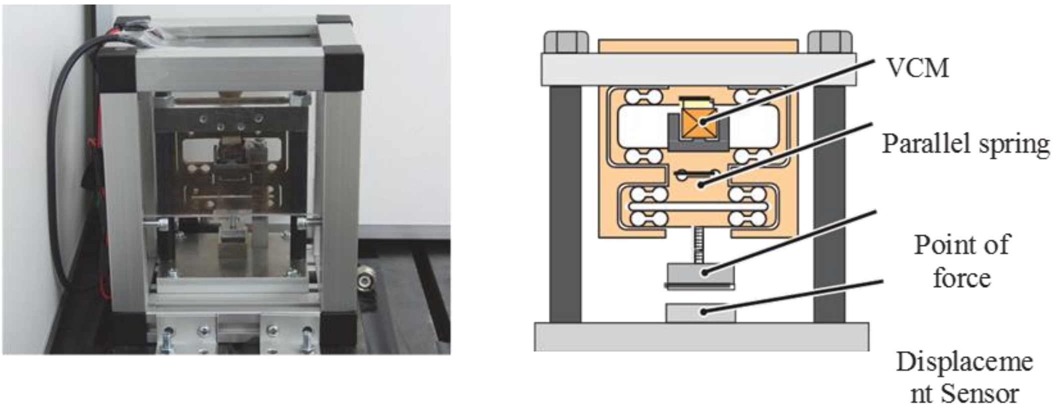
(L) $k_2>0$

Fig.12 Operations of zero-compliance mechanism



(a) Picture (b) Schematic drawing

Fig.13 Three-dimensional force measurement system using double series magnetic suspension



(a) Picture (b) Schematic drawing

Fig.14 Picture of single-dimensional force measurement module with a voice coil motor and parallel springs

The accuracy of measurement of the above-mentioned three-component force measurement apparatus was limited mainly because of some interaction among the four units. To solve such a problem, a new apparatus for 3-component force measurement is designed and fabricated [14]. Figure 15 shows a picture of this apparatus. It has a common detection point for three components. Force measurements have been carried out with the apparatus. The results show that 3-component forces can be measured with a high resolution by the developed apparatus.

Another apparatus for three-dimensional-force measurement has been developed as shown by Fig.16 [15]. In the previous apparatus shown by Fig.13, the point of force was fully suspended magnetically. Thereby, it rotated during measurement, which caused some measurement error. In the developed system, the point of force is suspended by leaf springs to prevent the rotation of the point of force. Therefore, higher measurement accuracy has been achieved by this measurement apparatus.

5. Conclusion

The zero-compliance mechanism has unique characteristics that cannot be achieved by conventional suspensions. Taking advantage of such characteristics, we have applied this mechanism to vibration isolation

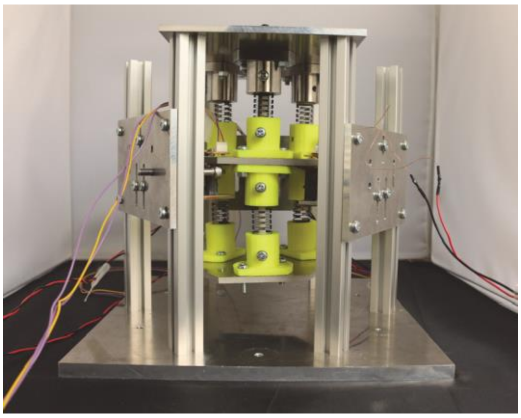
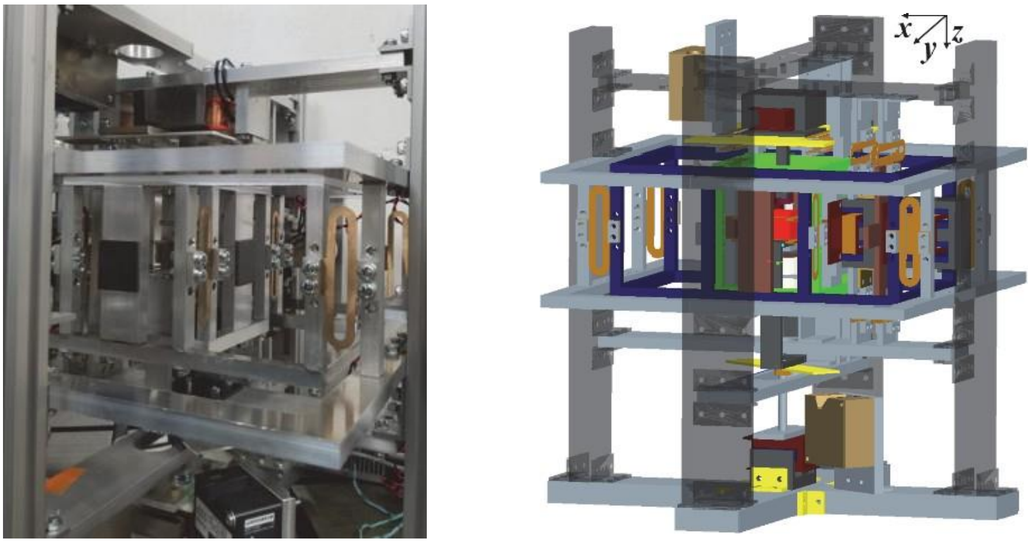


Fig.15 Picture of three-component force measurement apparatus with a single detection point



(b) Picture

(b) Schematic drawing

Fig.17 Three-dimensional force appratus with the point of force suspended by leaf springs and force measurement. Several apparatuses have been developed to demonstrate the efficacy of the zero- compliance mechanism in each case. Further research is still under way to increase its applications.

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